LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

FIFTH SEMESTER – NOVEMBER 2009

ST 5500 - ESTIMATION THEORY

Date & Time: 3/11/2009 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

Part-A

Answer All the questions **(10 x 2 = 20)**

1. **Define unbiasedness of an estimator with an example.**
2. **Give an example for a consistent estimator.**
3. **Explain sufficiency of an estimator.**
4. **Define UMVUE.**
5. **Show that {w (0, σ^2), $\sigma \rightarrow 0$} is not complete .**
6. **Mention any two properties of MLE.**
7. **Define prior distribution.**
8. **Define Risk function.**
9. **What is linear estimation?**
10. **Define BLUE.**

PART-B

Answer any FIVE questions **(5 x 8 = 40)**

11. **State and prove Cramer – Rao inequality.**
12. **If $X_1, X_2, \ldots, X_n$ is a random sample from a normal population N(μ, 1), then show that T = $\frac{1}{n}\sum_{1}^{n} X_i^2$ is an unbiased estimator of $\mu^2 + 1$.**
13. **Establish Rao – Blackwell theorem.**
14. **Let $X_1, X_2, \ldots, X_n$ be i.i.d. b(1, p) random variables. Show that T = $\sum_{1}^{n} X_i$. is a sufficient statistic and also complete for the parameter p.**
15. **Explain the method of moments in estimation.**
16. **A random sample $X_1, X_2, \ldots, X_n$ is taken from a normal population with mean zero and variance σ^2. Find MVUE for σ^2.**
17. **Explain the general procedure of Bayes estimation.**
18. **Obtain Least Square estimators for the parameters of a linear function y = α + βx.**

PART-C

Answer any two questions **(2 x 20 = 40)**

19. **a) State and prove Chapman – Robbin's inequality.**

b) Prove that for Cauchy's distribution not sample mean but sample median is a consistent estimator of the population mean.

20. **a) State and prove Lehmann – Scheffe theorem.**

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a uniform population on (0, θ). Find a sufficient estimator for θ.

21. **a) Explain Method of minimum Chi – Square and modified minimum Chi – Square.**

b) Estimate α and β in the following distribution by the method of moments.

$f(x; \alpha, \beta) = [\beta^{\alpha} / \sqrt{\alpha}\ (\alpha)]\ x^{\alpha - 1}\ e^{-\beta x}, 0 \leq x < \infty.$

22. **a) Obtain the maximum likelihood estimate of θ in $f(x; \theta) = (1 + \theta)\ x^{\theta}$, $0 \leq x \leq 1$, based on a random sample of size n. Examine whether it is sufficient for θ.**

b) Find Bayes estimator of the parameter p of a binomial distribution with x successes out of n given that the prior distribution of p is a beta distribution with parameters α and β.